NONLINEAR HARMONIC SELECTION IN AN FEL UNDULATOR SYSTEM

Sandra Biedron, Luca Giannessi*, Karen Horovitz, Stephen Milton
Electrical and Computer Engineering, Colorado State University, Fort Collins, CO 80523, USA
*ENEA C.R. Frascati Via E. Fermi 45, 00044 Roma, Italy

Abstract

Both fundamental and higher order harmonics are produced within the synchrotron radiation emitted during the free electron laser (FEL) process. These harmonics are not always wanted and at times can be detrimental to the overall operation of the FEL; furthermore, they can have a negative effect on user operations. Thus, the ability to control the harmonics is an important area of research. In this paper we discuss some possible means of controlling the harmonics and use PERSEO simulations to reduce the 5th harmonic of a test FEL system to acceptable levels.

CONCEPT

FEL Basics

In an FEL's undulator, the electron oscillations and its synchrotron radiation are phase matched, allowing energy exchange between the electrons and the EM radiation. This energy exchange causes the electrons to bunch at the resonant wavelength and subsequently emit coherently at this wavelength and its harmonics (Eqn. 1).

$$\lambda_{Rn} = \frac{\lambda_{und}}{2n\gamma^2}\left(1+K^2/2\right) \tag{1}$$

Here, λ_{und} is the period of the undulator, K is the normalized maximum field strength of the undulator magnet, n is the harmonic number, γ is the normalized electron beam energy, and λ_{Rn} is the output wavelength for harmonic number n ($n = 1$ implies the fundamental). K can be interpreted in a different manner. It is the ratio of the maximum angle of oscillation of the electron beam within the undulator field, $x_{\max}$, to the opening angle of the radiation, θ_r (Eqn. 2).

$$K = \frac{x_{\max}}{\theta_r} = x_{\max}\gamma \tag{2}$$

Harmonics

The primary origin of harmonics generated by a single particle can be seen graphically in Figure 1. As K gets larger than about 1, the radiation pulse gets richer in harmonics and more intense.

The strengths of the on-axis fundamental and harmonics generated for a single electron traveling through an undulator have been calculated [1] and are directly proportional to the following function,

$$F_n(K) = \frac{n^2K^2}{\xi^2}\left\{J_{\frac{n-1}{2}}\left[\frac{nK^2}{4\xi}\right] - J_{\frac{n+1}{2}}\left[\frac{nK^2}{4\xi}\right]\right\}^2 \tag{3}$$

$$(n = 1,3,5,\ldots) \qquad \xi = \left(1+K^2/2\right)$$

where n is the harmonic number and the J's are Bessel functions. The function $F_n(K)$ is plotted in Figure 2. It is clear that emission from a single particle is dominated by the harmonics for K values larger than about 1.5.

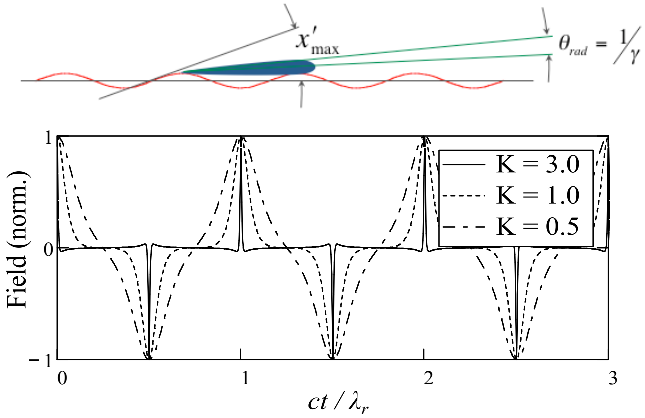

Figure 1: Graphical depiction of the impact of K on the harmonic content of the synchrotron radiation.

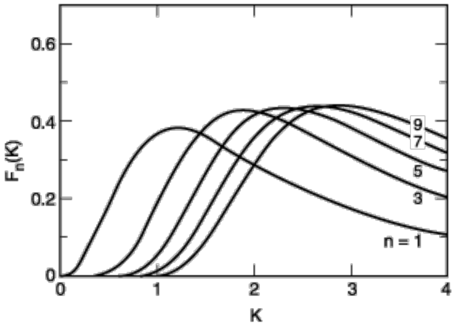

Figure 2: Strengths of the on-axis fundamental and harmonics as a function of K.

In an FEL we have coherent emission of a large number of electrons. Consider the case of a train of single electrons spaced exactly at the resonant wavelength of the undulator/beam system. The phases of fundamental and harmonic fields generated from the electron train along the direction of propagation overlap perfectly and a coherent build up of the field occurs. Adding N electrons to each of the original electrons in the train so that each one can be considered a microbunch of charge Ne increases the field in a coherent fashion. This would be the ideal final state of an FEL, each microbunch completely compressed onto a single point and each spaced precisely one resonant wavelength apart.

In an actual FEL the electrons within a microbunch are spread over an optical period (Fig 3). This spreading introduces phase differences between the electrons and thus the coherence between every particle is not exact, and since for the harmonics the microbunch is spread over a larger phase range the degradation of coherent emission becomes worse for the harmonics. Note also that the microbunching is being driven by the fundamental; however, because the bunching has spatial harmonics there is significant coherent harmonic emission, but not nearly at the levels implied by single particle emission as shown in figure 1. Saturation of the emission of the odd harmonics are typically two to three orders of magnitude less than the preceding harmonic, and even though this is significantly lower than the fundamental, these levels can represent problems for high average power systems.

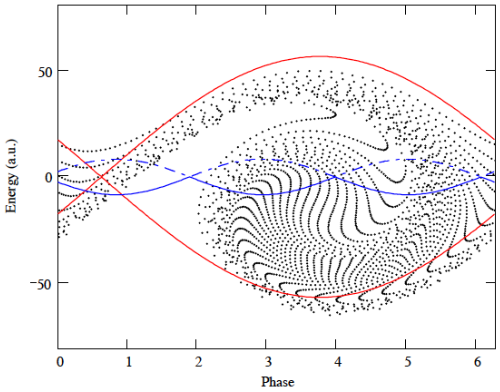

Figure 3: Longitudinal phase space plot of the FEL microbunching including the computed sepratrices for the fundamental (red) and 3rd harmonic (blue).

The harmonics can be a useful feature or can be harmful. In the former, one is able to generate wavelengths significantly shorter than the fundamental, and in the past it has been shown how to enhance the power contained in the harmonics [2-4]. However, harmonics can actually damage the downstream optics and/or interrupt user experiments, at times making them undesirable [5]. We have shown using multi-harmonic undulators how to reduce them [6]. Here we explore other methods.

Cancellation of Harmonic Emission

We have shown previously that it is possible to enhance the harmonic content of the FEL in order to reach higher harmonic powers at shorter wavelengths. In the past using optical materials (i.e. gratings) to separate out the unwanted harmonics was used but there is still a chance of damage and leakage, thus disrupting users. As a test case we explore reducing these higher harmonics.

Deliberate, sudden phase shifts between the radiation and e-beam microbunching can damp out unwanted harmonics [7, 8]. Here, we will focus on cancellation of the 5th harmonic. Just prior to saturation we apply a phase shift of plus or minus $\pi/5$ (relative to the fundamental). This can be done by simply placing an appropriately sized drift between two undulator sections. As the beam is already microbunched it will once again begin to radiate coherently in the following undulator, and although there will be a small phase error between the fundamental radiation and the fundamental component of the microbunching, the growth of the fundamental, while still in the exponential regime, should not be significantly impacted. On the other hand, the shift places the 5th harmonic component of the microbunching exactly π out of phase with the existing 5th harmonic radiation. The resulting emission from the microbunching thus cancels out the existing emission.

Unfortunately, the new 5th harmonic radiation will initially cancel what is already there, but then will start growing again as the beam is still microbunched with a significant amount of harmonic content at the 5th harmonic. In addition, harmonic suppression based on phase shifts is effective if the harmonic field and the electron-bunching factor are perfectly phase matched before applying the shift. Saturation in an FEL amplifier occurs because of a phase de-synchronization. Therefore harmonic suppression relying on phase shifts becomes less effective at saturation, where the efficiency of the FEL is higher.

From the basic theory the nonlinear harmonics of an FEL have a growth rate equal to n times the fundamental growth rate, where n is the harmonic number. We can accurately predict how much distance is required to cancel the existing radiation and how much it takes for the 5th harmonic signal to reach its original intensity. Then one applies another $\pi/5$ phase shift, this time with the opposite sign to the original one, thus allowing the beam to cancel once again the existing 5th harmonic radiation. This process can be repeated however many times as needed to allow the fundamental to saturate while keeping the 5th harmonic power low.

Optimizing the Fundamental Emission

A second possibility of reducing harmonic growth is to optimize the FEL for emission only at the fundamental. We can design an undulator system where the initial part has a high K value and then a later section that has a K value of roughly 0.8 to 0.9, to reduce harmonics.

Initially the FEL process would proceed as expected. Microbunching would occur at the fundamental and the harmonic content would grow and start radiating coherently. Near saturation but still at a point when the unwanted 5th harmonic strength is tolerably low , we switch to an undulator with a lower K value. The fundamental would continue to grow; however, as there is no significant emission at the 5th harmonic, even though the beam has spatial 5th harmonic content there is little or no harmonic emission as K is at a value that harmonic emission is not significant and the fundamental dominate. The system then reaches saturation at the fundamental with harmonic levels much lower than for a single high K value undulator system.

However, there is a price to pay. The gain lengths are significantly shorter at high K values. Switching to a lower K value implies a longer gain length and a longer undulator system. This can be minimized by keeping the K value large up to the point that the unwanted harmonic starts growing rapidly and then making the switch.

Combining Both Methods

Ideally we would start with a high K value followed by an undulator of lower K value, while still maintaining the resonant condition. At the same time the drift between the two sections could be chosen to provide a π/n phase shift and stop the growth of the n^{th} harmonic while possibly reducing the existing n^{th} harmonic radiation.

SIMULATION

We set out to prove the above reasoning by use of simulations. While there are many different configurations for FELs, in these simulations, we will consider the case where we have an amplifier configuration, i.e., an external laser is used to seed the FEL process.

PERSEO Description

The PERSEO FEL-cad (MathCAD) library was written by one of the authors and allows the simulation of a wide variety of FEL configurations [9, 10]. Functions for the generation of phase space variables, for the solution of the pendulum-like equation and for manipulating the phase space in a number of devices are available. These function can be combined in order to model more complicated situations such as time dependent simulations, 3D simulations, oscillator FEL configurations, optical klystron, cascaded FELs, et cetera. A Mathcad Worksheet for 1D includes correction for the 3D filling factor and emittance induced inhomogeneous broadenings and 3D versions have been tested. PERSEO also includes higher order harmonics and startup from shot-noise.

Simulation Results

We performed the following set of simulations. 1) No phase shifts or K changes (Figure 4). 2) Same as 1; however, a phase shift is performed prior to the system reaching saturation. 3) Same as 1; however, multiple phase shifts are performed prior to the system reaching saturation. 4) Same as 1; however, the K value of the undulator is changed and made significantly smaller prior to the system reaching saturation. Nominal parameters are given in Table 1 for all simulations.

Figure 4 shows the nominal case where the K value is held constant and there are no phase shifts. Typical behavior is seen and the 5th harmonic peak power saturating just over at a bit over 1 kW.

Table 1: FEL Simulation Parameters

Electron Beam Parameter	**Value**
Energy (MeV)	100
Energy Spread (%)	0.15
Norm. X/Y Emit. (mm mrad)	10.0
Peak Current (A)	200
Seed Laser Parameter	**Value**
Seed Wavelength (nm)	1040
Peak seed Intensity (W/cm^2)	400×10^3
Peak seed Power (W)	1005
Undulator Parameter	**Value**
Period (cm)	2.54
K (nominal)	2.07
Length (m)	5.7
Wavelength (nm)	1040
Photon Energy (eV)	1.2
Periods	226
Twiss beta (m)	0.765

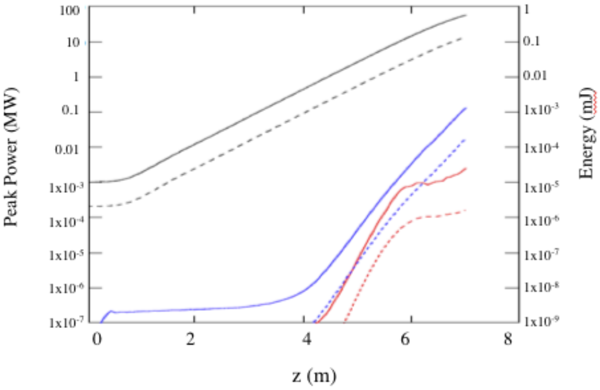

Figure 4: Nominal case. No phase shifts, no K variation. Black: Fundamental, Blue: 3rd Harmonic, Red: 5th Harmonic. Solid: Peak Power, Dashed: Energy.

Figure 5 shows the case where a $\pi/5$ phase shift is inserted just prior to saturation, i.e. at roughly 5 m. As expected, the 5th harmonic intensity drops quickly, but within about 2 gain lengths begins, once again, to rise rapidly. Saturation of the 5th harmonic now occurs at a slightly lower value than in the nominal case, but certainly not an order of magnitude or more. We attribute this slight reduction to the slight reduction seen on the fundamental caused by the $\pi/5$ phase shift.

We next tried multiple $\pm\pi/5$ phase shifts in an attempt to counter the growth of the 5th harmonic. The results are shown in Figure 6. As can be seen in the figure, even with judicious choice of the placement of the phase shifts it was not possible to push the 5th harmonic saturated power down by much more than was done with a single-phase shift. We attribute this to the fact that phase shifts of constant amplitude ($\pm\pi/5$) become less effective approaching saturation. In addition, the discreet nature of the phase shifts vs. the natural spread in phase of the spatial 5th harmonic of the beam may play a role. Even though one is successful in cancelling out radiation from a part of the beam there are still others that can radiate coherently.

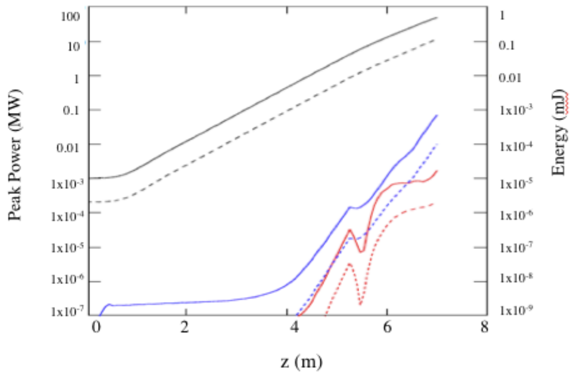

Figure 5: Similar to the nominal case, but with a $\pi/5$ phase shift inserted at roughly the 5-m point.

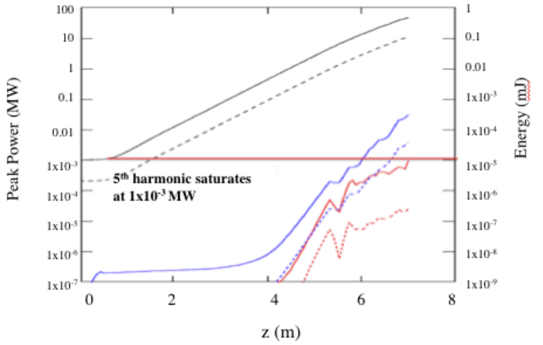

Figure 6: Similar to the nominal case, but with a $\pi/5$ phase shift inserted at roughly the 5-m point and repeated $\pm\pi/5$ phase shifts inserted as per the text.

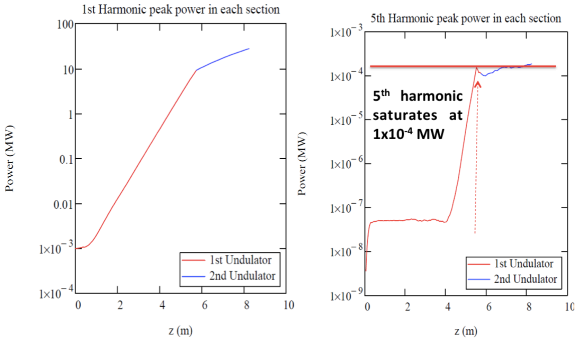

Figure 7: Saturation is a magnitude of one order less than without phase shift

In the final simulation we allow the FEL power to grow within the undulator, but prior to saturation, and at a 5th harmonic power level that is acceptably low, we make a sudden decrease in K to a value of 1 and at the same time adjust the period to 5.31 cm to remain resonant with the fundamental. The results are shown in Figure 7. The 5th harmonic power stops increasing following the decrease in K while the fundamental power continues to grow albeit at a slower growth rate. Saturation of the 5th harmonic is now at a level at least 1 order of magnitude down from the nominal case.

CONCLUSIONS

Decreasing the power in the nonlinear harmonics of FELs can prove essential to applications that require high fundamental power but are susceptible to significant powers in the higher harmonics. In this case one needs to develop means to either remove the generated harmonics or to inhibit or reverse the harmonic growth process. We have devised two such schemes. One is to make periodic shifts in the relative phase between the beam and the resultant electromagnetic wave. The other is to stop further growth of the harmonics by decreasing the K value of the undulator system. It was found that in the first case that one can have a small impact on the harmonics, but this method seems to be limited. In the second method one can stop further increase on the harmonics, but the expense one pays is in a slightly longer undulator necessary to reach full saturation of the fundamental. This second method is simple and appears to be able to limit the growth of the harmonics to acceptable values.

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